(D1) **Binary Pulsar**

Through systematic searches during the past decades, astronomers have found a large number of millisecond pulsars (spin period < 10 ms). The majority of these pulsars are found in binaries, with nearly circular orbits.

For a pulsar in a binary orbit, the measured pulsar spin period (P) and the measured line-of-sight acceleration (a) both vary systematically due to the orbital motion. For circular orbits, this variation can be described mathematically in terms of orbital phase angle ϕ ($0 \leq \phi \leq 2\pi$) as,

$$P(\phi) = P_0 + P_t \cos\phi \qquad \text{where } P_t = \frac{2\pi P_0 r}{c P_B}$$

$$a(\phi) = -a_t \sin\phi \qquad \text{where } a_t = \frac{4\pi^2 r}{P_B^2}$$

where P_B is the orbital period of the binary, P_0 is the intrinsic spin period of the pulsar and r is the radius of the orbit.

The following table gives one such set of measurements of P and a at different heliocentric epochs (moments in time), T, expressed in truncated Modified Julian Days (tMJD), i.e. number of days since MJD $=$ 2,440,000.

No.	T (tMJD)	P (μs)	a (m s^{-2})
1	5740.654	7587.8889	- 0.92 ± 0.08
2	5740.703	7587.8334	- 0.24 ± 0.08
3	5746.100	7588.4100	- 1.68 ± 0.04
4	5746.675	7588.5810	+ 1.67 ± 0.06
5	5981.811	7587.8836	+ 0.72 ± 0.06
6	5983.932	7587.8552	- 0.44 ± 0.08
7	6005.893	7589.1029	+ 0.52 ± 0.08
8	6040.857	7589.1350	+ 0.00 ± 0.04
9	6335.904	7589.1358	+ 0.00 ± 0.02

By plotting $a(\phi)$ as a function of $P(\phi)$, we can obtain a curve. As evident from the relations above, this curve in the period-acceleration plane is an ellipse.

By an analysis of this data set, we estimate the intrinsic spin period, P_0, the orbital period, P_B, and the orbital radius, r, assuming a circular orbit.

(D1.1) Plot the data, including error bars, in the period-acceleration plane (mark your graph as "D1.1"). **7**

(D1.2) Draw an ellipse that is a best fit to the data (on the same graph "D1.1"). **2**

(D1.3) From the plot, estimate P_0, P_t and a_t, including their corresponding uncertainties. **7**

(D1.4) Write expressions for P_B and r in terms of P_0, P_t, a_t. **4**

(D1.5) Calculate an approximate value of P_B and r based on your estimations made in (D1.3), including uncertainties. **6**

(D1.6) Calculate the orbital phase angle, ϕ, corresponding to the epochs of the following five observations in the above table: No.s 1, 4, 6, 8, 9. **4**

(D1.7) Refine the estimate of the orbital period, P_B, using the results in part (D1.6) in the following way:

(D1.7a) First determine the initial epoch, T_0, which corresponds to the nearest epoch of zero orbital phase angle, before the first observation. **2**

(D1.7b) The time expected, T_{calc}, from the calculated orbital phase angle of each observation is given by, **7**

$$T_{\text{calc}} = T_0 + \left(n + \frac{\phi}{360°}\right) P_{\text{B}},$$

where n is the number of full cycles that elapse between T_0 and T (or T_{calc}). Estimate n and T_{calc} for each of the five observations in part (D1.6). Write down the differences $T_{\text{O}-\text{C}}$ between the observed T and T_{calc} and enter these in the table in the Summary Answersheet.

(D1.7c) Plot $T_{\text{O}-\text{C}}$ against n (mark your graph as "D1.7"). **4**

(D1.7d) Determine the refined values of the initial epoch, $T_{0,\text{r}}$, and the orbital period, $P_{\text{B},\text{r}}$. **7**

(D2) Distance to the Moon

Geocentric ephemerides of the Moon for September 2015 are given in the form of a table. Each reading was taken at 00:00 UT.

Date	R.A. (α)			Dec. (δ)			Angular Size (θ)	Phase (ϕ)	Elongation
	h	m	s	°	'	"	"		of the Moon
Sep 01	0	36	46.02	3	6	16.8	1991.2	0.927	148.6° W
Sep 02	1	33	51.34	7	32	26.1	1974.0	0.852	134.7° W
Sep 03	2	30	45.03	11	25	31.1	1950.7	0.759	121.1° W
Sep 04	3	27	28.48	14	32	4.3	1923.9	0.655	107.9° W
Sep 05	4	23	52.28	16	43	18.2	1896.3	0.546	95.2° W
Sep 06	5	19	37.25	17	55	4.4	1869.8	0.438	82.8° W
Sep 07	6	14	19.23	18	7	26.6	1845.5	0.336	70.7° W
Sep 08	7	7	35.58	17	23	55.6	1824.3	0.243	59.0° W
Sep 09	7	59	11.04	15	50	33.0	1806.5	0.163	47.5° W
Sep 10	8	49	0.93	13	34	55.6	1792.0	0.097	36.2° W
Sep 11	9	37	11.42	10	45	27.7	1780.6	0.047	25.1° W
Sep 12	10	23	57.77	7	30	47.7	1772.2	0.015	14.1° W
Sep 13	11	9	41.86	3	59	28.8	1766.5	0.001	3.3° W
Sep 14	11	54	49.80	0	19	50.2	1763.7	0.005	7.8° E
Sep 15	12	39	50.01	-3	20	3.7	1763.8	0.026	18.6° E
Sep 16	13	25	11.64	-6	52	18.8	1767.0	0.065	29.5° E
Sep 17	14	11	23.13	-10	9	4.4	1773.8	0.120	40.4° E
Sep 18	14	58	50.47	-13	2	24.7	1784.6	0.189	51.4° E
Sep 19	15	47	54.94	-15	24	14.6	1799.6	0.270	62.5° E
Sep 20	16	38	50.31	-17	6	22.8	1819.1	0.363	73.9° E
Sep 21	17	31	40.04	-18	0	52.3	1843.0	0.463	85.6° E
Sep 22	18	26	15.63	-18	0	41.7	1870.6	0.567	97.6° E
Sep 23	19	22	17.51	-17	0	50.6	1900.9	0.672	110.0° E
Sep 24	20	19	19.45	-14	59	38.0	1931.9	0.772	122.8° E
Sep 25	21	16	55.43	-11	59	59.6	1961.1	0.861	136.2° E
Sep 26	22	14	46.33	-8	10	18.3	1985.5	0.933	150.0° E
Sep 27	23	12	43.63	-3	44	28.7	2002.0	0.981	164.0° E
Sep 28	0	10	48.32	0	58	58.2	2008.3	1.000	178.3° E
Sep 29	1	9	5.89	5	38	54.3	2003.6	0.988	167.4° W
Sep 30	2	7	39.02	9	54	16.1	1988.4	0.947	153.2° W

The composite image[1] below shows multiple snapshots of the Moon taken at different times during the total lunar eclipse, which occurred in this month. To take this image, the north-south line of the umbra is tracked.

For this problem, assume that the observer is at the centre of the Earth and that angular size refers to the angular diameter of the object or shadow.

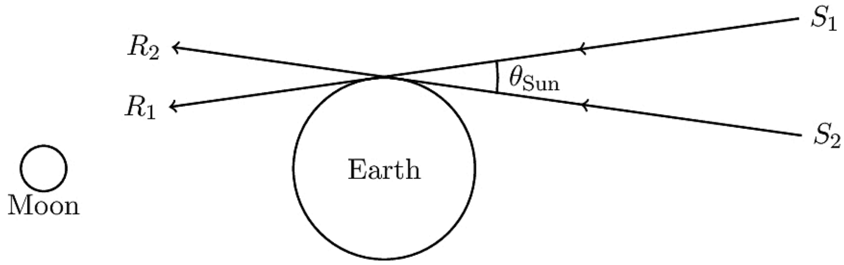
(D2.1) In September 2015, the apogee of the lunar orbit is closest to the:
New Moon / First Quarter / Full Moon / Third Quarter.
Tick the correct answer in the Summary Answersheet. No justification for your answer is necessary. **3**

(D2.2) In September 2015, the ascending node of the lunar orbit with respect to the ecliptic is closest to the:
New Moon / First Quarter / Full Moon / Third Quarter.
Tick the correct answer in the Summary Answersheet. No justification for your answer is necessary. **4**

(D2.3) Estimate the eccentricity, e, of the lunar orbit from the given data. **4**

(D2.4) Estimate the angular size of the umbra, θ_{umbra}, in terms of the angular size of the Moon, θ_{Moon}. Show your working on the image given on the back of the Summary Answersheet. **8**

(D2.5) The angle subtended by the Sun at the Earth, on the day of the lunar eclipse, is known to be $\theta_{\text{Sun}} = 1915.0''$. In the figure below, S_1R_1 and S_2R_2 are rays coming from diametrically opposite sides of the solar disk. The figure is not to scale. **9**



Calculate the angular size of the penumbra, θ_{penumbra}, in terms of θ_{Moon}. Assume that the observer is at the centre of the Earth.

(D2.6) Let θ_{Earth} be the angular size of the Earth as seen from the centre of the Moon. Calculate the angular size of the Moon, θ_{Moon}, as seen from the centre of the Earth, on the eclipse day in terms of θ_{Earth}. **5**

(D2.7) Estimate the radius of the Moon, R_{Moon}, in km from the results above. **3**

(D2.8) Estimate the shortest distance, r_{perigee}, and the furthest distance, r_{apogee}, to the Moon. **4**

(D2.9) Use appropriate data from September 10 to estimate the Earth-Sun distance, d_{Sun}. **10**

[1] Credit: NASA's Scientific Visualization Studio

(D3) **Type IA Supernovae**

Supernovae of type Ia are considered very important for the measurements of large extragalactic distances. The brightening and subsequent dimming of these explosions follow a characteristic light curve, which helps in identifying these as supernovae type Ia.

Light curves of all type Ia supernovae can be fitted by the same model light curve, if they are scaled appropriately. In order to achieve this, we first have to compute the light curves in the reference frame of the host galaxy. This can be done by considering that all observed time intervals, Δt_{obs}, have been cosmologically stretched by a factor of (1+z). The time interval in the rest frame of the host galaxy is denoted by Δt_{gal}.

The rest frame light curve of a supernova changes by two magnitudes, compared to the peak magnitude, in a time interval, Δt_0, following the time of the peak. If we further scale the time intervals by a factor of s (i.e. $\Delta t_s = s\Delta t_{\text{gal}}$) such that the scaled value of Δt_0 is the same for all supernovae, the light curves turn out to have the same shape. It also turns out that s is related linearly to the absolute magnitude, M_{peak}, at the peak luminosity for the supernova. That is, we can write

$$s = a + bM_{\text{peak}} ,$$

where a and b are constants. Knowing the scaling factor, one can determine absolute magnitudes of supernovae at unknown distances applying the above linear equation.

The table below contains data for three supernovae, including their distance moduli, μ (shown for the first two only), their recession speeds, cz, and their apparent magnitudes, m_{obs}, at different times. The time $\Delta t_{\text{obs}} \equiv t - t_{\text{peak}}$ shows the number of days from the date at which the respective supernova reached peak brightness. The observed magnitudes have already been corrected for interstellar as well as atmospheric extinction.

Name	**SN2006TD**	**SN2006IS**	**SN2005LZ**
μ (mag)	**34.27**	**35.64**	
cz (km s^{-1})	**4515**	**9426**	**12060**
Δt_{obs} (days)	m_{obs} (mag)	m_{obs} (mag)	m_{obs} (mag)
-15.00	19.41	18.35	20.18
-10.00	17.48	17.26	18.79
-5.00	16.12	16.42	17.85
0.00	15.74	16.17	17.58
5.00	16.06	16.41	17.72
10.00	16.72	16.82	18.24
15.00	17.53	17.37	18.98
20.00	18.08	17.91	19.62
25.00	18.43	18.39	20.16
30.00	18.64	18.73	20.48

(D3.1) Calculate Δt_{gal} values for all three supernovae, and fill in the empty boxes in the data tables on the BACK side of the Summary Answersheet. On a sheet of graph paper, plot the points and draw the three light curves in their rest frames (mark your graph as "D3.1"). **15**

(D3.2) Take the scaling factor, s_2, for the supernova SN2006IS to be 1.00. Calculate the scaling factors, s_1 and s_3, for the other two supernovae SN2006TD and SN2005LZ, respectively, by measuring Δt_0 for each of them. **5**

(D3.3) Compute the scaled time differences, Δt_s, for all three supernovae. Write the values for Δt_s in the same data table on the Summary Answersheet. On another sheet of graph paper, plot the points and draw 3 light curves to verify that they now have an identical profile (mark your graph as "D3.3"). **14**

(D3.4) Calculate the absolute magnitudes at peak brightness, $M_{\text{peak},1}$, for SN2006TD and $M_{\text{peak},2}$, for SN2006IS. Use these values to calculate a and b. **[6]**

(D3.5) Calculate the absolute magnitude at peak brightness, $M_{\text{peak},3}$, and distance modulus, μ_3, for SN2005LZ. **[4]**

(D3.6) Use the distance modulus μ_3 to estimate the value of the Hubble constant, H_0. In addition, estimate the characteristic age of the universe, T_{H}. **[6]**