(D1) **Binary Pulsar**

Through systematic searches during the past decades, astronomers have found a large number of millisecond pulsars (spin period < 10 ms). Majority of these pulsars are found in binaries, with nearly circular orbits.

For a pulsar in a binary orbit, the measured pulsar spin period (P) and the measured line-of-sight acceleration (a) both vary systematically due to orbital motion. For circular orbits, this variation can be described mathematically in terms of orbital phase ϕ $(0 \le \phi \le 2\pi)$ as,

$$P(\phi) = P_0 + P_{\mathrm{t}} \cos\phi \qquad \text{where } P_t = \frac{2\pi P_0 r}{c P_{\mathrm{B}}}$$

$$a(\phi) = -a_{\mathrm{t}} \sin\phi \qquad \text{where } a_{\mathrm{t}} = \frac{4\pi^2 r}{P_{\mathrm{B}}^2}$$

where P_{B} is the orbital period of the binary, P_0 is the intrinsic spin period of the pulsar and r is the radius of the orbit.

The following table gives one such set of measurements of P and a at different heliocentric epochs, T, expressed in truncated Modified Julian Days (tMJD), i.e. number of days since MJD $= 2{,}440{,}000$.

No.	T (tMJD)	P (μs)	a (m s^{-2})
1	5740.654	7587.8889	- 0.92 ± 0.08
2	5740.703	7587.8334	- 0.24 ± 0.08
3	5746.100	7588.4100	- 1.68 ± 0.04
4	5746.675	7588.5810	+ 1.67 ± 0.06
5	5981.811	7587.8836	+ 0.72 ± 0.06
6	5983.932	7587.8552	- 0.44 ± 0.08
7	6005.893	7589.1029	+ 0.52 ± 0.08
8	6040.857	7589.1350	+ 0.00 ± 0.04
9	6335.904	7589.1358	+ 0.00 ± 0.02

By plotting $a(\phi)$ as a function of $P(\phi)$, we can obtain a parametric curve. As evident from the relations above, this curve in the period-acceleration plane is an ellipse.

In this problem, we estimate the intrinsic spin period, P_0, the orbital period, P_{B}, and the orbital radius, r, by an analysis of this data set, assuming a circular orbit.

(D1.1) Plot the data, including error bars, in the period-acceleration plane (mark your graph as "D1.1"). **7**

(D1.2) Draw an ellipse that appears to be a best fit to the data (on the same graph "D1.1"). **2**

(D1.3) From the plot, estimate P_0, P_{t} and a_{t}, including error margins. **7**

(D1.4) Write expressions for P_{B} and r in terms of P_0, P_{t}, a_{t}. **4**

(D1.5) Calculate approximate value of P_{B} and r based on your estimations made in (D1.3), including error margins. **6**

(D1.6) Calculate orbital phase, ϕ, corresponding to the epochs of the following five observations in the above table: data rows 1, 4, 6, 8, 9. **4**

(D1.7) Refine the estimate of the orbital period, P_{B}, using the results in part (D1.6) in the following way:

(D1.7a) First determine the initial epoch, T_0, which corresponds to the nearest epoch of zero orbital phase before the first observation. **2**

(D1.7b) The expected time, T_{calc}, of the estimated orbital phase angle of each observation is given by, **7**

$$T_{\mathrm{calc}} = T_0 + \left(n + \frac{\phi}{360^\circ}\right) P_{\mathrm{B}},$$

where n is the number of full cycle of orbital phases that may have elapsed between T_0 and T (or T_{calc}). Estimate n and T_{calc} for each of the five observations in part (D1.6). Note down difference $T_{\text{O}-\text{C}}$ between observed T and T_{calc}. Enter these calculations in the table given in the Summary Answersheet.

(D1.7c) Plot $T_{\text{O}-\text{C}}$ against n (mark your graph as "D1.7"). **4**

(D1.7d) Determine the refined values of the initial epoch, $T_{0,\text{r}}$, and the orbital period, $P_{\text{B,r}}$. **7**

(D2) **Distance to the Moon**

Geocentric ephemerides of the Moon for September 2015 are given in the form of a table. Each reading was taken at 00:00 UT.

Date	R.A. (α)			Dec. (δ)			Angular Size (θ)	Phase (ϕ)	Elongation
	h	m	s	°	'	"	"		Of Moon
Sep 01	0	36	46.02	3	6	16.8	1991.2	0.927	148.6° W
Sep 02	1	33	51.34	7	32	26.1	1974.0	0.852	134.7° W
Sep 03	2	30	45.03	11	25	31.1	1950.7	0.759	121.1° W
Sep 04	3	27	28.48	14	32	4.3	1923.9	0.655	107.9° W
Sep 05	4	23	52.28	16	43	18.2	1896.3	0.546	95.2° W
Sep 06	5	19	37.25	17	55	4.4	1869.8	0.438	82.8° W
Sep 07	6	14	19.23	18	7	26.6	1845.5	0.336	70.7° W
Sep 08	7	7	35.58	17	23	55.6	1824.3	0.243	59.0° W
Sep 09	7	59	11.04	15	50	33.0	1806.5	0.163	47.5° W
Sep 10	8	49	0.93	13	34	55.6	1792.0	0.097	36.2° W
Sep 11	9	37	11.42	10	45	27.7	1780.6	0.047	25.1° W
Sep 12	10	23	57.77	7	30	47.7	1772.2	0.015	14.1° W
Sep 13	11	9	41.86	3	59	28.8	1766.5	0.001	3.3° W
Sep 14	11	54	49.80	0	19	50.2	1763.7	0.005	7.8° E
Sep 15	12	39	50.01	-3	20	3.7	1763.8	0.026	18.6° E
Sep 16	13	25	11.64	-6	52	18.8	1767.0	0.065	29.5° E
Sep 17	14	11	23.13	-10	9	4.4	1773.8	0.120	40.4° E
Sep 18	14	58	50.47	-13	2	24.7	1784.6	0.189	51.4° E
Sep 19	15	47	54.94	-15	24	14.6	1799.6	0.270	62.5° E
Sep 20	16	38	50.31	-17	6	22.8	1819.1	0.363	73.9° E
Sep 21	17	31	40.04	-18	0	52.3	1843.0	0.463	85.6° E
Sep 22	18	26	15.63	-18	0	41.7	1870.6	0.567	97.6° E
Sep 23	19	22	17.51	-17	0	50.6	1900.9	0.672	110.0° E
Sep 24	20	19	19.45	-14	59	38.0	1931.9	0.772	122.8° E
Sep 25	21	16	55.43	-11	59	59.6	1961.1	0.861	136.2° E
Sep 26	22	14	46.33	-8	10	18.3	1985.5	0.933	150.0° E
Sep 27	23	12	43.63	-3	44	28.7	2002.0	0.981	164.0° E
Sep 28	0	10	48.32	0	58	58.2	2008.3	1.000	178.3° E
Sep 29	1	9	5.89	5	38	54.3	2003.6	0.988	167.4° W
Sep 30	2	7	39.02	9	54	16.1	1988.4	0.947	153.2° W

The composite graphic[1] below shows multiple snapshots of the Moon taken at different times during the total lunar eclipse, which occurred in this month. For each shot, the centre of frame was coinciding with the central north-south line of umbra.

For this problem, assume that the observer is at the centre of the Earth and angular size refers to angular diameter of the object / shadow.

[1] Credit: NASA's Scientific Visualization Studio

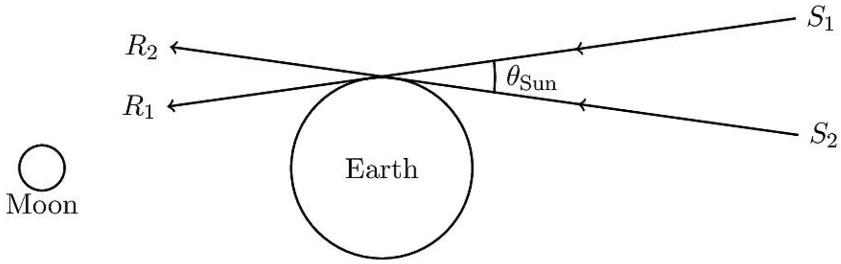
(D2.1) In September 2015, apogee of the lunar orbit is closest to New Moon / First Quarter / Full Moon / Third Quarter. Tick the correct answer in the Summary Answersheet. No justification for your answer is necessary. **3**

(D2.2) In September 2015, the ascending node of lunar orbit with respect to the ecliptic is closest to New Moon / First Quarter / Full Moon / Third Quarter. Tick the correct answer in the Summary Answersheet. No justification for your answer is necessary. **4**

(D2.3) Estimate the eccentricity, e, of the lunar orbit from the given data. **4**

(D2.4) Estimate the angular size of the umbra, θ_{umbra}, in terms of the angular size of the Moon, θ_{Moon}. Show your working on the image given on the backside of the Summary Answersheet. **8**

(D2.5) The angle subtended by the Sun at Earth on the day of the lunar eclipse is known to be $\theta_{\mathrm{Sun}} = 1915.0''$. In the figure below, S_1R_1 and S_2R_2 are rays coming from diametrically opposite ends of the solar disk. The figure is not to scale. **9**



Calculate the angular size of the penumbra, $\theta_{\mathrm{penumbra}}$, in terms of θ_{Moon}. Assume the observer to be at the centre of the Earth.

(D2.6) Let θ_{Earth} be angular size of the Earth as seen from the centre of the Moon. Calculate the angular size of the Moon, θ_{Moon}, as would be seen from the centre of the Earth on the eclipse day in terms of θ_{Earth}. **5**

(D2.7) Estimate the radius of the Moon, R_{Moon}, in km from the results above. **3**

(D2.8) Estimate the shortest distance, r_{perigee}, and the farthest distance, r_{apogee}, to the Moon. **4**

(D2.9) Use appropriate data from September 10 to estimate the distance, d_{Sun}, to the Sun from the Earth. **10**

(D3) **Type IA Supernovae**

Supernovae of type Ia are considered very important for the measurements of large extragalactic distances. The brightening and subsequent dimming of these explosions follow a characteristic light curve, which helps in identifying these as supernovae of type Ia.

Light curves of all type Ia supernovae can be fit to the same model light curve, when they are scaled appropriately. In order to achieve this, we first have to express the light curves in the reference frame of the host galaxy by taking care of the cosmological stretching/dilation of all observed time intervals, Δt_{obs}, by a factor of $(1 + z)$. The time interval in the rest frame of the host galaxy is denoted by Δt_{gal}.

The rest frame light curve of a supernova changes by two magnitudes compared to the peak in a time interval Δt_0 after the peak. If we further scale the time intervals by a factor of s (i.e. $\Delta t_s = s\Delta t_{\text{gal}}$) such that the scaled value of Δt_0 is the same for all supernovae, the light curves turn out to have the same shape. It also turns out that s is related linearly to the absolute magnitude, M_{peak}, at the peak luminosity for the supernova. That is, we can write

$$s = a + bM_{\text{peak}} ,$$

where a and b are constants. Knowing the scaling factor, one can determine absolute magnitudes of supernovae at unknown distances from the above linear equation.

The table below contains data for three supernovae, including their distance moduli, μ (for the first two), their recession speed, cz, and their apparent magnitudes, m_{obs}, at different times. The time $\Delta t_{\text{obs}} \equiv t - t_{\text{peak}}$ shows number of days from the date at which the respective supernova reached peak brightness. The observed magnitudes have already been corrected for interstellar as well as atmospheric extinction.

Name	**SN2006TD**	**SN2006IS**	**SN2005LZ**
μ (mag)	**34.27**	**35.64**	
cz (km s^{-1})	**4515**	**9426**	**12060**
Δt_{obs} (days)	m_{obs} (mag)	m_{obs} (mag)	m_{obs} (mag)
-15.00	19.41	18.35	20.18
-10.00	17.48	17.26	18.79
-5.00	16.12	16.42	17.85
0.00	15.74	16.17	17.58
5.00	16.06	16.41	17.72
10.00	16.72	16.82	18.24
15.00	17.53	17.37	18.98
20.00	18.08	17.91	19.62
25.00	18.43	18.39	20.16
30.00	18.64	18.73	20.48

(D3.1) Compute Δt_{gal} values for all three supernovae, and fill them in the given blank boxes in the data tables on the BACK side of the Summary Answersheet. On a graph paper, plot the points and draw the three light curves in the rest frame (mark your graph as “D3.1”). **15**

(D3.2) Take the scaling factor, s_2, for the supernova SN2006IS to be 1.00. Calculate the scaling factors, s_1 and s_3, for the other two supernovae SN2006TD and SN2005LZ, respectively, by calculating Δt_0 for them. **5**

(D3.3) Compute the scaled time differences, Δt_s, for all three supernovae. Write the values for Δt_s in the same data tables on the Summary Answersheet. On another graph paper, plot the points and draw the 3 light curves to verify that they now have an identical profile (mark your graph as “D3.3”). **14**

(D3.4) Calculate the absolute magnitudes at peak brightness, $M_{\text{peak},1}$, for SN2006TD and $M_{\text{peak},2}$, for SN2006IS. Use these values to calculate a and b. **6**

(D3.5) Calculate the absolute magnitude at peak brightness, $M_{\text{peak},3}$, and distance modulus, μ_3, for SN2005LZ. **4**

(D3.6) Use the distance modulus μ_3 to estimate the value of Hubble's constant, H_0. Further, estimate the characteristic age of the universe, T_{H}. **6**